Arithmetic and Geometric Sequences and Series

Arithmetic Sequence

(a) A sequence of numbers T(1), T(2), T(3), ……, T(n), is called an **arithmetic sequence** if :

$$d = T(2) - T(1) = T(3) - T(2) = \dots = T(n) - T(n-1)$$

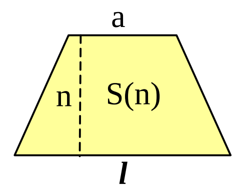
where d is a constant known as the **common difference**.

(b) The first term = **a** = T(1)

The last term = ***l*** = T(n)

(c) The **general term** : $T(n) = a + (n - 1)d$

The **sum** of the first n terms : $S(n) = \frac{n}{2}(a + \boldsymbol{l})$ (see diagram)



$$S(n) = \frac{n}{2}\left[2a + (n-1)d\right]$$

(d) If a, b and c are any three consecutive terms of an arithmetic sequence, the middle term b is called the **arithmetic mean** of a and c, then $b = \frac{a+c}{2}$

Geometric Sequence

(a) A sequence of numbers T(1), T(2), T(3), ……, T(n), is called an **geometric sequence** if :

$$r = \frac{T(2)}{T(1)} = \frac{T(3)}{T(2)} = \cdots = \frac{T(n)}{T(n-1)}$$

where r is a constant known as the **common ratio.**

(b) The **general term** : $T(n) = ar^{n-1}$

The **sum** of the first n terms : $S(n) = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}$, where $r \neq 1$.

(c) The **sum to infinity** of an infinite geometric series :

$S(\infty) = \frac{a}{1-r}$, where r must be in the range: $-1 < r < 1$.

(d) If a, b and c are any three consecutive terms of an geometric sequence, the middle term b is called the **geometric mean** of a and c, then $b = \pm\sqrt{ac}$