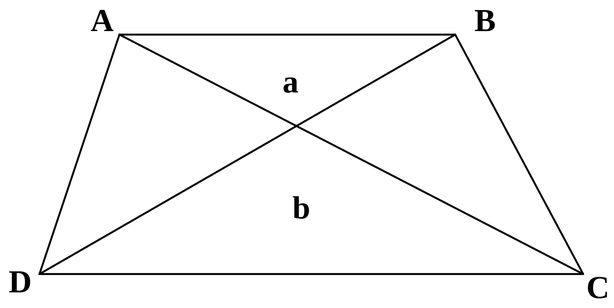
Area

Question 1

A trapezium ABCD with AB//DC is divided into four triangles by its diagonals.

Let the triangles adjacent to the parallel sides have areas a and b.

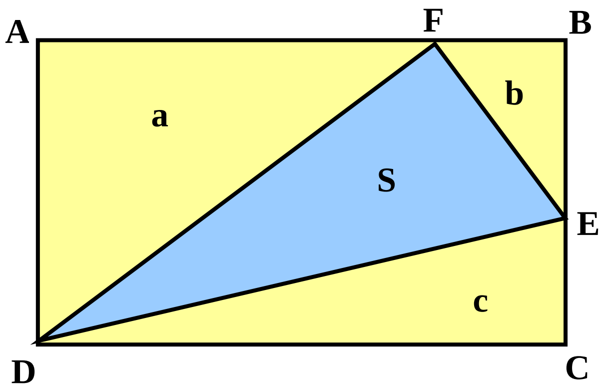
Find the area of the trapezoid in terms of a and b.



Question 2

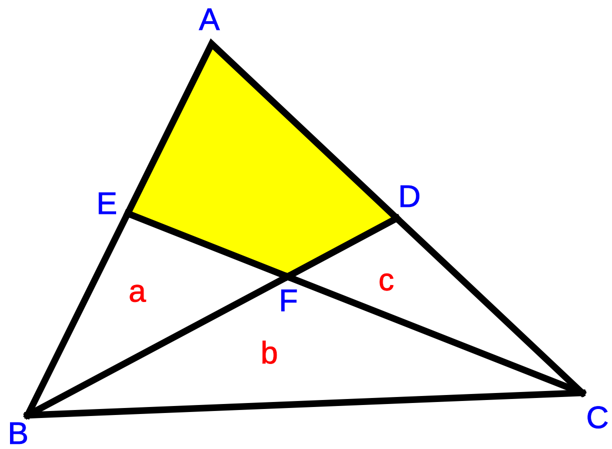
ABCD is a rectangle.
The areas of the right angled triangles are a, b, c, as in the figure.

Find the area of the triangle, S, in terms of a, b, c.



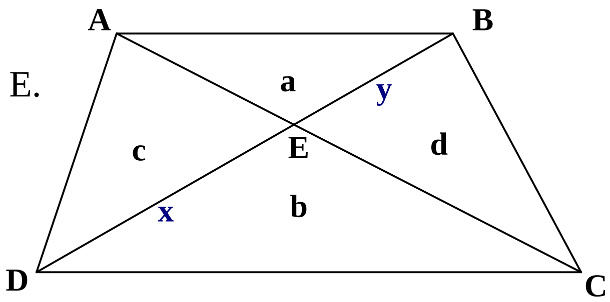
Question 3

ABC is a triangle.
BD and CE cut at F.
If area of $\Delta BEF = a$, area of $\Delta BFC = b$, area of $\Delta CFD = c$, find the area of the quadrilateral AEFD.



Answers

Question 1



Let the two diagonals AC and BD meets at E.

Let DE = x, BE = y

c : a = x : y,   b : d = x : y

$\therefore$ c : a = b : d

$\therefore$ ab = cd (1)

Area of ΔACD = Area of ΔBCD

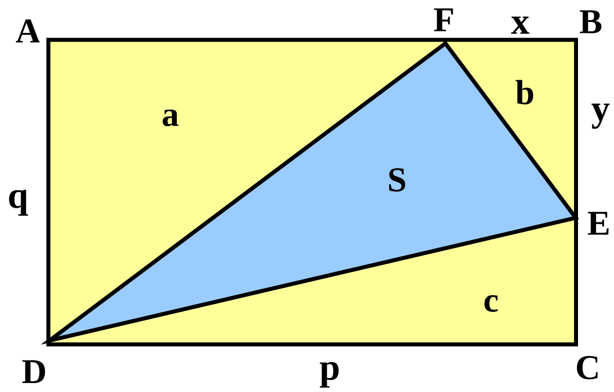
$\therefore$ c + b = d + b

$\therefore$ c = d (2)

(2) ↓ (1), $c = d = \sqrt{ab}$

$\therefore$ Area of trapezium ABCD = a + b + c + d $= a + b + 2\sqrt{ab} = \underline{\underline{\left(\sqrt{a} + \sqrt{b}\right)^2}}$

Question 2



Let

p = DC, q = AD
x = FB, y = BE

$$x = p - \frac{2a}{q}, \quad y = q - \frac{2c}{p}$$

$$b = \frac{1}{2}xy = \frac{1}{2}\left(p - \frac{2a}{q}\right)\left(q - \frac{2c}{p}\right)$$

$$2b = pq - 2a - 2c - \frac{4ac}{pq}$$

$$pq - 2(a+b+c) - \frac{4ac}{pq} = 0$$

$(pq)^2 - 2(a + b + c)(pq) - 4ac = 0$

Consider only the positive root, we have:

$$pq = \frac{2(a+b+c) + \sqrt{[2(a+b+c)]^2 - 4(1)(-4ac)}}{2(1)} = (a+b+c) + \sqrt{(a+b+c)^2 + 4ac}$$

$\therefore$ S = pq – (a + b + c) $= \underline{\underline{\sqrt{(a+b+c)^2 + 4ac}}}$

Question 3

$$\frac{a+x}{y}=\frac{BF}{FD}=\frac{b}{c}$$

$\therefore$ by = ca + cx (1)

$$\frac{c+y}{x}=\frac{CF}{FE}=\frac{b}{a}$$

$\therefore$ bx = ac + ay (2)

Solve (1), (2), we have:

$$x=\frac{ac(a+b)}{b^2-ac},\quad y=\frac{ac(b+c)}{b^2-ac}$$

$\therefore$ Area of quad. AEFD = x + y $=\underline{\underline{\dfrac{ac(a+2b+c)}{b^2-ac}}}$