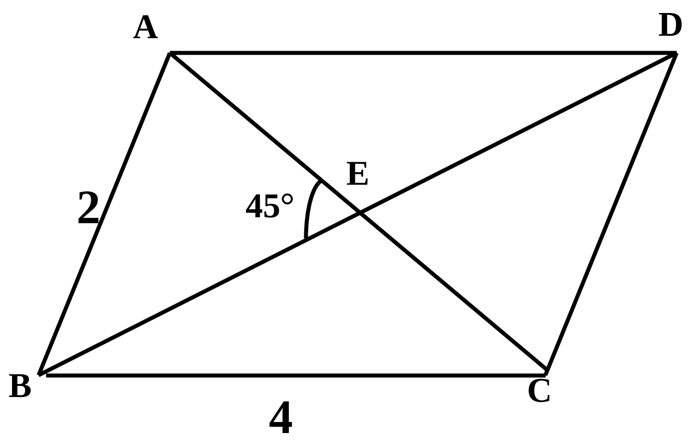
Trigonometry

Question

1. **In the parallelogram ABCD, AB = 2, BC = 4, the angle between the diagonals AC and BD is 45°.**

   Find the area of ABCD.



2. **(a) Prove that if $\tan x \tan(A - x) = k$**

   then $\cos(2x - A) = \dfrac{1+k}{1-k}\cos A$

   (b) Hence solve the equation:

$$\left(2-\sqrt{3}\right)\tan x \tan\left(60^{\circ} - x\right) = 1$$

Solution

1. **Let AE = x, BE = y**

In ΔAEB,

$$AB^2 = AE^2 + BE^2 - 2(AE)(BE)\cos\angle AEB$$

i.e. $2^2 = x^2 + y^2 - 2xy\cos 45^\circ$

$$4 = x^2 + y^2 - \sqrt{2}xy \qquad \text{......... (1)}$$

In ΔBEC, $16 = x^2 + y^2 + \sqrt{2}xy$ .... **(2)**

(2) – (1), $12 = 2\sqrt{2}xy$

$$xy = 3\sqrt{2} \qquad \text{......... (3)}$$

Area of ABCD = 4 × Area ΔAEB $= 4 \times \frac{1}{2} xy\sin 45^\circ = \underline{\underline{6}}$

2\. (a) $\tan x \tan(A - x) = k$

$$\frac{\sin x \sin(A-x)}{\cos x \cos(A-x)} = \frac{k}{1}$$

$$\frac{\sin x \sin(A-x) + \cos x \cos(A-x)}{\sin x \sin(A-x) - \cos x \cos(A-x)} = \frac{k+1}{k-1}$$

$$\frac{\cos[x-(A-x)]}{-\cos[x+(A-x)]} = \frac{k+1}{k-1}$$

$$\frac{\cos(2x-A)}{-\cos A} = \frac{k+1}{k-1}$$

$$\therefore \ \cos(2x-A) = \frac{1+k}{1-k}\cos A$$

> Use Componendo et Dividendo,
>
> $$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

(b) $(2-\sqrt{3})\tan x \tan(60^\circ - x) = 1$

$$\tan x \tan(60^\circ - x) = \frac{1}{2-\sqrt{3}} = 2 + \sqrt{3}$$

Put A = 60°, k = $2+\sqrt{3}$

By (a), $\cos(2x-60^\circ) = \frac{1+(2+\sqrt{3})}{1-(2+\sqrt{3})}\cos 60^\circ$

$$\cos(2x-60^\circ) = \frac{(3+\sqrt{3})}{-(1+\sqrt{3})}\frac{(1-\sqrt{3})}{(1-\sqrt{3})}\frac{1}{2}$$

$$\cos(2x-60^\circ) = -\frac{\sqrt{3}}{2}$$

2x - 60° = 360°n ± 150°, where n is an integer.

$\therefore$ **x = 180°n + 105° or 180°n - 45°, where n is an integer.**