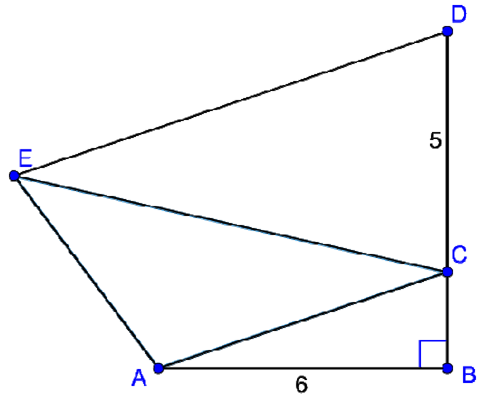
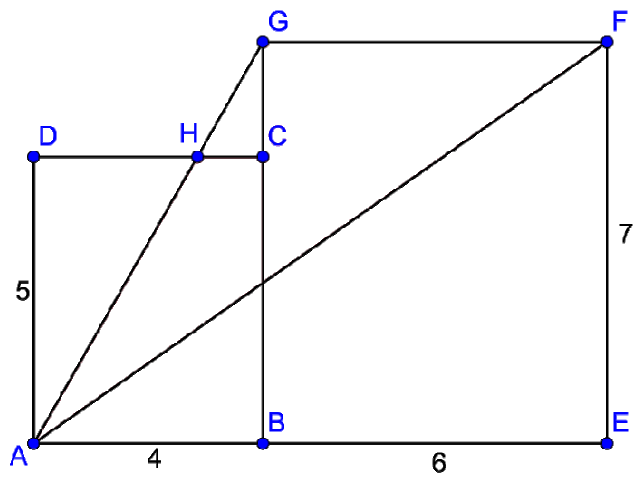
Easier Area of plane figures

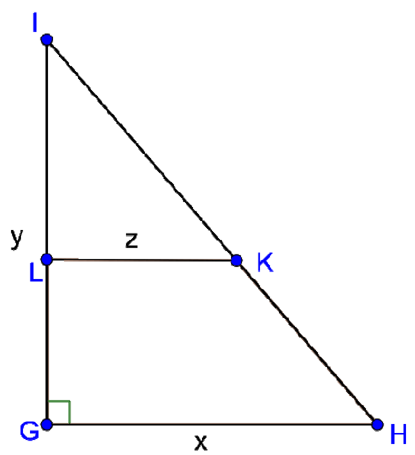
1. If $AC \parallel EB, \angle ABD = 90°, AB = 6, CD = 5$, find the area of ΔACE.



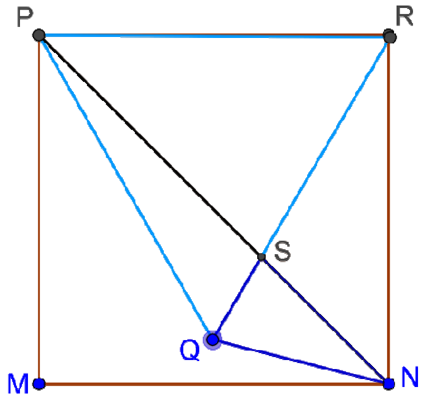
2. Two rectangles $ABCD, BEFG$ of dimensions 4×5, 6×7 respectively are given. Find the shaded area $AFGCH$.



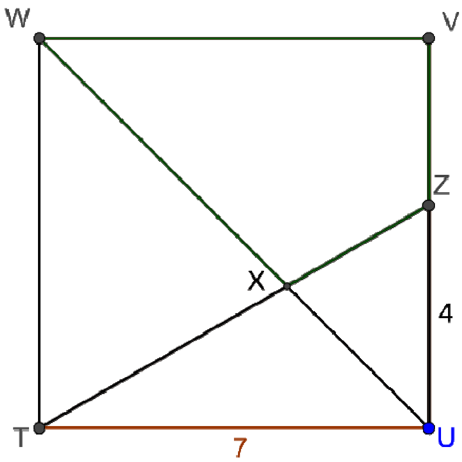
3. If $GH \parallel LK$, $\angle IGH = 90°, GH = x, GI = y, LK = z$, find the area of trapezium $GHKL$ in terms of x, y and z .



4. If ΔPQR is equilateral and $PMNR$ is a square. Each side of the square is of unit 1, PM cuts QR at S, find the area of ΔQNS.



5. If $TUVW$ is a square of sides 7, $UZ = 4$, WU cuts TZ at X, find the area of $VWXZ$.



6. **(Quickie)** Given a triangle ΔABC right angled at A . If $BC = 11$ and the altitude from A to BC is 6 . Find the area of the triangle.

Denote $[\Delta\mathbf{ABC}]$ **be the area of** $\Delta\mathbf{ABC}$ **.**

1. $[\Delta ACE] = [\Delta ACD]$, same base AC , same height between parallel lines.

$$= \frac{1}{2} \times 5 \times 6 = \underline{\underline{\mathbf{15}}}$$

2. $CG = 7 - 5 = 2$

$\Delta HCG \sim \Delta ABG \Longrightarrow \frac{HC}{AB} = \frac{GC}{GB} \Longrightarrow \frac{HC}{4} = \frac{2}{7} \Longrightarrow HC = \frac{8}{7}$

$[\Delta HCG] = \frac{1}{2} \times 2 \times \frac{8}{7} = \frac{8}{7}$

$[AFGC] = [\Delta AFG] - [\Delta HCG] = \frac{1}{2} \times 4 \times 7 - \frac{8}{7} = \underline{\underline{\mathbf{\frac{90}{7}}}}$

3. Let $LG = w$

$\Delta ILK \sim \Delta IGH \Longrightarrow \frac{IL}{IG} = \frac{LK}{GH} \Longrightarrow \frac{y-w}{y} = \frac{z}{x} \Longrightarrow w = y\left(\frac{x-z}{x}\right)$

$[GHKL] = \frac{1}{2} \times (x+z) \times y\left(\frac{x-z}{x}\right) = \frac{y(x+z)(x-z)}{2x}$

4. $PM = MN = NP = RP = PQ = QR = 1$

$\angle PRQ = 60°, \angle SRN = 30°, \angle SNR = 45°$

Method 1	**Method 2**
Let h be the length of the perpendicular from S to NR. $\frac{h}{\tan 30°} + \frac{h}{\tan 45°} = 1$ $h\sqrt{3} + h = 1$ $h = \frac{1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{2}$ $[\Delta SNR] = \frac{1}{2} \times 1 \times (\sqrt{3}-1) = \frac{\sqrt{3}-1}{4}$	$\angle RSN = 180° - 30° - 45° = 105°$ By Sine Law on ΔSNR, $\frac{SR}{\sin 45°} = \frac{1}{\sin 105°}$ $SR = \frac{\sin 45°}{\sin 105°} = \frac{\sin 45°}{\sin 75°} = \frac{\sin 45°}{\sin(45°+30°)}$ $= \frac{\sin 45°}{\sin 45° \cos 30° + \cos 45° \sin 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\frac{1}{2}}$ $= \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$ $[\Delta SNR] = \frac{1}{2} SR \times RN \sin \angle SRN$ $= \frac{1}{2}(\sqrt{3}-1)\sin 30° = \frac{\sqrt{3}-1}{4}$

$\therefore [\Delta SQN] = [\Delta SQN] - [\Delta SNR] = \frac{1}{2} \times 1 \times 1 \times \sin 30° - \frac{\sqrt{3}-1}{4} = \frac{2-\sqrt{3}}{4}$

5. $[\Delta VWZ] = \frac{1}{2} \times 3 \times 7 = \frac{21}{2}$

$\Delta XWT \sim \Delta XUZ \Rightarrow \frac{TX}{XZ} = \frac{WT}{UZ} \Rightarrow \frac{TX}{XZ} = \frac{7}{4} \Rightarrow \frac{[\Delta XTW]}{[\Delta XZW]} = \frac{7}{4} \Rightarrow \begin{cases} [\Delta XTW] = 7k \\ [\Delta XZW] = 4k \end{cases}$

$[\Delta TWZ] = \frac{1}{2} \times 7 \times 7 = \frac{49}{2}$

$$[\Delta XTW] + [\Delta XZW] = [\Delta TWZ] \Rightarrow 7k + 4k = \frac{49}{2}$$

$$k = \frac{49}{22}$$

$$[\Delta XZW] = 4k = \frac{98}{11}$$

$[VWXZ] = [\Delta XZW] + [\Delta VWZ] = \frac{98}{11} + \frac{21}{2} = \mathbf{\frac{427}{22}}$

6. The answer is **not** $\frac{1}{2} \times 11 \times 6 = 33$ **.**

Draw a circle with BC as diameter. Then point A is a point on the circumference of the circle. (The theorem Angle in semi-circle ensures $\angle BAC$ is right angle.) The maximum altitude drawn from A to BC is then the radius of the circle in which ΔABC is isosceles.

However, the radius is $\frac{11}{2} = 5.5 < 6$. So there is **no possible solution**.