**Mooncakes**

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**(a)** A square box of side $d$ contains 4 moon-cakes each of radius $r$ is shown in the right diagram.

 Find the area shaded in yellow in terms of $d$ .

**(b)** **(i)** A rectangular moon-cake box contains two moon-cakes, each of radius r.

 A diagonal is drawn as shown in the right.

 Find the total area of the parts shaded yellow in terms of r.

** (ii)** Find the total area of the parts shaded yellow in terms of r.

 (Note that a small piece of area in yellow in the lower left corner is removed)

**(a)** The area shaded in yellow $=\frac{4}{16}\left(d^{2}-4r^{2}\right)=\frac{1}{4}\left[d^{2}-4\left(\frac{d}{4}\right)^{2}\right]=\overline{\overline{\frac{3 d^{2}}{16}}}$

**(b) (i)**



 Total area shaded in yellow = total area shaded in green

 Therefore, total area shaded in yellow

$$=\frac{1}{2}\left[area of rectangle-2\left(area of one circle\right)\right]$$

$$=\frac{1}{2}\left[\left(4r\right)\left(2r\right)-2\left(πr^{2}\right)\right]=4r^{2}-πr^{2}=\overline{\overline{\left(4-π\right)r^{2}}}$$

**(b)** **(ii)** We concentrate on the left square and find the lower left area that is removed.

 (in green)



 Area in green = area of ΔABC – area in yellow – area in orange

 Area of ΔABC $=\frac{1}{2}×2r×r=r^{2}$

 Area in yellow $=\frac{1}{4}×\left(area of square-area of circle\right)$

 $=\frac{1}{4}×\left(4r^{2}-πr^{2}\right)=\frac{1}{4}\left(4-π\right)r^{2}$

 $∠BAC=θ$ , then $∠OCD=∠ODC=θ$, $∠DOC=π-2θ$

 $\tan(θ=\frac{CB}{AB}=\frac{r}{2r})=\frac{1}{2}$

 Also by Pythagoras Theorem, $AC=\sqrt{5}r$, $\sin(θ=\frac{r}{\sqrt{5}r})=\frac{1}{\sqrt{5}}, \cos(θ)=\frac{2r}{\sqrt{5}r}=\frac{2}{\sqrt{5}}$

 Area of ΔDOC $=\frac{1}{2}r^{2}\sin(\left(π-2θ\right))=\frac{1}{2}r^{2}\sin(2θ=\frac{1}{2}r^{2}\left(2 sin θ cos θ\right))$

 $=\frac{1}{2}r^{2}\left(2 ×\frac{1}{\sqrt{5}}× \frac{2}{\sqrt{5}}\right)=\frac{2}{5}r^{2}$

 Area of segment in orange = Area of sector – area of ΔDOC

 $=\frac{1}{2}r^{2}\left(π-2θ\right)-\frac{2}{5}r^{2}=\frac{1}{2}πr^{2}-\frac{2}{5}r^{2}-r^{2}θ=\frac{1}{2}πr^{2}-\frac{2}{5}r^{2}-r^{2}tan^{-1}\frac{1}{2}$

 Area in green = area of ΔABC – area in yellow – area in orange

 $r^{2}-\frac{1}{4}\left(4-π\right)r^{2}-\left(\frac{1}{2}πr^{2}-\frac{2}{5}r^{2}-r^{2}tan^{-1}\frac{1}{2}\right)=\frac{2 r^{2}}{5}-\frac{π r^{2}}{4}+r^{2}tan^{-1}\frac{1}{2}$

 Lastly, the total area of the parts shaded yellow

= area in part (a) - missing small piece of area in green calculated above

 $=\left(4-π\right)r^{2}-\left[\frac{2 r^{2}}{5}-\frac{π r^{2}}{4}+r^{2}tan^{-1}\frac{1}{2}\right]$

 $=\overline{\overline{\frac{18 r^{2}}{5}-\frac{3π r^{2}}{4}-r^{2}tan^{-1}\frac{1}{2}}}$

**Yue Kwok Choy**

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